1　式の計算

1

(1)

$$与式=(3x-1)\{x(3x-1)+1\}$$
$$=x(3x-1)^2+3x-1$$
$$=x(9x^2-6x+1)+3x-1$$
$$=9x^3-6x^2+4x-1$$

(2)

$$与式=(2x-1)\cdot 3y-(2x-1)\cdot 4$$
$$=(2x-1)(3y-4)$$

(3)

$$与式=(3x)^3-4^3$$
$$=(3x-4)\{(3x)^2+3x\cdot 4+4^2\}$$
$$=(3x-4)(9x^2+12x+16)$$

(4)

$$与式=3\sqrt{2}-3\sqrt{3}+\sqrt{\frac{24}{3}}+\sqrt{\frac{24}{2}}$$
$$=3\sqrt{2}-3\sqrt{3}+\sqrt{8}+\sqrt{12}$$
$$=3\sqrt{2}-3\sqrt{3}+2\sqrt{2}+2\sqrt{3}$$
$$=5\sqrt{2}-\sqrt{3}$$

2

(1)

$$与式=\{(2x+3y)+(2x-3y)\}^3-3(2x+3y)(2x-3y)\{(2x+3y)+(2x-3y)\}$$
$$=(4x)^3-3\{(2x)^2-(3y)^2\}\cdot 4x$$
$$=64x^3-12x(4x^2-9y^2)$$
$$=16x^3+108xy^2$$

補足

$A^3+B^3=(A+B)^3-3AB(A+B)$ を用いた

(2)

$$与式=\{(a^2+9)+3a\}\{(a^2+9)-3a\}$$
$$=(a^2+9)^2-(3a)^2$$
$$=a^4+18a^2+81-9a^2$$
$$=a^4+9a^2+81$$

(3)

$$
\begin{aligned}
与式 &= \{(x-1)(x^2+x+1)\}^2 \\
&= (x^3-1)^2 \\
&= (x^3)^2 - 2x^3 + 1 \\
&= x^6 - 2x^3 + 1
\end{aligned}
$$

3

(1)

$$与式 = (2x+3y)(4x-3y)$$

補足

たすきがけ

(2)

$$
\begin{aligned}
与式 &= \{(x^2+x)-2\}\{(x^2+x)-6\} \\
&= (x^2+x-2)(x^2+x-6) \\
&= (x-1)(x+2)(x-2)(x+3)
\end{aligned}
$$

(3)

$$
\begin{aligned}
与式 &= (b^2-c^2)a - bc^2 - b^2c \\
&= (b+c)(b-c)a - bc(b+c) \\
&= (b+c)\{(b-c)a - bc\} \\
&= (b+c)(ab-bc-ca)
\end{aligned}
$$

補足

a の 1 次式，b の 2 次式，c の 2 次式だから，a の式の次数が最も小さい。

よって，a について整理し，因数分解するのが楽。

(4)

$$
\begin{aligned}
与式 &= (2x)^2 - y^2 - 2y - 1 \\
&= (2x)^2 - (y^2+2y+1) \\
&= (2x)^2 - (y+1)^2 \\
&= \{2x+(y+1)\}\{2x-(y+1)\} \\
&= (2x+y+1)(2x-y-1)
\end{aligned}
$$

4

(1)

$$
\begin{aligned}
与式 &= \{(\sqrt{2}+\sqrt{3})+\sqrt{5}\}\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\} \\
&= (\sqrt{2}+\sqrt{3})^2 - (\sqrt{5})^2 \\
&= 2 + 2\sqrt{6} + 3 - 5 \\
&= 2\sqrt{6}
\end{aligned}
$$

(2)

$$与式=\frac{1}{\sqrt{5}+\sqrt{3}}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$=\frac{\sqrt{5}-\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\sqrt{3}-1}{\left(\sqrt{3}\right)^2-1^2}$$
$$=\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}$$
$$=\frac{\sqrt{5}-1}{2}$$

(3)

$$与式=\frac{\left(\sqrt{17}-4\right)^2+\left(\sqrt{17}+4\right)^2}{\left(\sqrt{17}+4\right)\left(\sqrt{17}-4\right)}$$
$$=\frac{17-8\sqrt{17}+16+\left(17+8\sqrt{17}+16\right)}{17-16}$$
$$=66$$

5

(1)

$$与式=\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(a^2+c^2+2ac\right)-\left(a^2+b^2+c^2-2ab-2bc+2ac\right)$$
$$=a^2+b^2+c^2$$

(2)

与式$=\left(4a^2-1\right)\left(a^2-a+4\right)$より， a^2の項は$4a^2\times4+(-1)\times a^2=15a^2$ $\therefore 15$

6

$$A+B=6x^2+2xy=2x(3x+y)$$
$$A-B=6xy-10y^2=2y(3x-5y)$$
$$A^2-B^2=(A+B)(A-B)=4xy(3x+y)(3x-5y)$$

7

(1)

$$与式=\{(x-1)(x-7)\}\{(x-3)(x-5)\}+15$$
$$=\left\{\left(x^2-8x\right)+7\right\}\left\{\left(x^2-8x\right)+15\right\}+15$$
$$=\left(x^2-8x\right)^2+22\left(x^2-8x\right)+120$$
$$=\left\{\left(x^2-8x\right)+12\right\}\left\{\left(x^2-8x\right)+10\right\}$$
$$=\left(x^2-8x+12\right)\left(x^2-8x+10\right)$$
$$=(x-2)(x-6)\left(x^2-8x+10\right)$$

(2)

$$\begin{aligned}
与式 &= 2x^2+(3y+1)x-2y^2+7y-3\\
&= 2x^2+(3y+1)x-(2y-1)(y-3)\\
&= \{x+(2y-1)\}\{2x-(y-3)\}\\
&= (x+2y-1)(2x-y+3)
\end{aligned}$$

(3)

$$\begin{aligned}
与式 &= a^2b-ab^2+bc(b-c)+c^2a-ca^2\\
&= (b-c)a^2-(b^2-c^2)a+bc(b-c)\\
&= (b-c)a^2-(b-c)(b+c)a+bc(b-c)\\
&= (b-c)\{a^2-(b+c)a+bc\}\\
&= (b-c)(a-b)(a-c)\\
&= -(a-b)(b-c)(c-a)
\end{aligned}$$

(4)

$$\begin{aligned}
与式 &= (x^4-1)-18x^2\\
&= \{(x^2-1)^2+2x^2\}-18x^2\\
&= (x^2-1)^2-16x^2\\
&= (x^2-1)^2-(4x)^2\\
&= \{(x^2-1)+4x\}\{(x^2-1)-4x\}\\
&= (x^2+4x-1)(x^2-4x-1)
\end{aligned}$$

8

(1)

$$\begin{aligned}
与式 &= \frac{4}{(1+\sqrt{2})+\sqrt{3}}\\
&= \frac{4}{(1+\sqrt{2})+\sqrt{3}}\cdot\frac{(1+\sqrt{2})-\sqrt{3}}{(1+\sqrt{2})-\sqrt{3}}\\
&= \frac{4(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-3}\\
&= \frac{4(1+\sqrt{2}-\sqrt{3})}{(1+2+2\sqrt{2})-3}\\
&= \frac{4(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}}\\
&= \sqrt{2}(1+\sqrt{2}-\sqrt{3})\\
&= \sqrt{2}+2-\sqrt{6}\\
&= 2+\sqrt{2}-\sqrt{6}
\end{aligned}$$

(2)

（ア）

$$
\begin{aligned}
与式 &= \sqrt{28 + 2\cdot 5\sqrt{3}} \\
&= \sqrt{28 + 2\sqrt{75}} \\
&= \sqrt{\left(\sqrt{25} + \sqrt{3}\right)^2} \\
&= \sqrt{\left(5 + \sqrt{3}\right)^2} \\
&= 5 + \sqrt{3}
\end{aligned}
$$

（イ）

$$
\begin{aligned}
与式 &= \sqrt{\frac{54 - 2\cdot 7\sqrt{5}}{2}} \\
&= \frac{\sqrt{54 - 2\sqrt{49\cdot 5}}}{\sqrt{2}} \\
&= \frac{\sqrt{\left(\sqrt{49} - \sqrt{5}\right)^2}}{\sqrt{2}} \\
&= \frac{\sqrt{\left(7 - \sqrt{5}\right)^2}}{\sqrt{2}} \\
&= \frac{7 - \sqrt{5}}{\sqrt{2}} \\
&= \frac{7\sqrt{2} - \sqrt{10}}{2}
\end{aligned}
$$

(3)

$$
\begin{aligned}
P &= \sqrt{\left(a^2 + 1\right) + 2a} + \sqrt{\left(a^2 + 1\right) - 6a + 8} \\
&= \sqrt{a^2 + 2a + 1} + \sqrt{a^2 - 6a + 9} \\
&= \sqrt{(a + 1)^2} + \sqrt{(a - 3)^2} \\
&= |a + 1| + |a - 3|
\end{aligned}
$$

よって，

$a < -1$ のとき

$P = \{-(a + 1)\} + \{-(a - 3)\} = -2a + 2$

$-1 \leq a < 3$ のとき

$P = (a + 1) + \{-(a - 3)\} = 4$

$3 \leq a$ のとき

$P = (a + 1) + (a - 3) = 2a - 2$

9

$$
\begin{aligned}
与式 &= \{a+(b+c)\}\{a^2-(b+c)a+b^2+c^2-bc\} \\
&= \{a+(b+c)\}\{a^2-(b+c)a+(b+c)^2-3bc\} \\
&= a^3-(b+c)a^2+(b+c)^2a-3abc+(b+c)a^2-(b+c)^2a+(b+c)^3-3bc(b+c) \\
&= a^3-3abc+\{(b+c)^3-3bc(b+c)\} \\
&= a^3-3abc+b^3+c^3 \\
&= a^3+b^3+c^3-3abc
\end{aligned}
$$

$$
\begin{aligned}
与式 &= (2x)^3+(3y)^3+(-1)^3-3\cdot 2x\cdot 3y\cdot(-1) \\
&= \{2x+3y+(-1)\}\{(2x)^2+(3y)^2+(-1)^2-2x\cdot 3y-3y\cdot(-1)-(-1)\cdot 2x\} \\
&= (2x+3y-1)(4x^2+9y^2+1-6xy+3y+2x) \\
&= (2x+3y-1)(4x^2-6xy+9y^2+2x+3y+1)
\end{aligned}
$$